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THE LAMINAR WALL PLUME IN A TRANSVERSE MAGNETIC FIELD

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NOMENCLATURE

B, magnetic field strength;
B_0, reference magnetic field strength;
C_p, specific heat at constant pressure;
f, dimensionless similarity stream function;
g, acceleration due to gravity;
I, dimensionless heat flux integral;
$\dot{m}$, vertical mass flow rate;
N, constant introduced in equation (8);
n, exponent introduced in equation (8);
Q, heat source strength per length;
Pr, Prandtl number;
u, vertical velocity;
v, horizontal velocity;
x, vertical coordinate;
y, horizontal coordinate;
Z_L, Lykoudis number.

Greek symbols

α, coefficient of thermal expansion;
η, similarity variable;
θ, temperature excess above undisturbed ambient;
θ_w, wall temperature excess above undisturbed ambient;
κ, thermal diffusivity;
μ, dynamic viscosity;
ν, kinematic viscosity;
ρ, fluid density;
σ, electrical conductivity;
τ, wall shear stress;
ϕ, normalized temperature similarity function;
ψ, stream function.

INTRODUCTION

THIS work examines the effect of a horizontal magnetic field on the steady laminar plume rising from a horizontal line heat source on a vertical adiabatic wall. One motivation for this study is the possibility that magneto fluid mechanic wall plumes may need to be accounted for in nuclear fusion reactor blankets. Some conceptual designs call for a blanket consisting of a nominally stagnant body of liquid metal or molten salt traversed by coolant-carrying tubes. These would generate negatively buoyant line plumes in a region of strong magnetic fields. Although the present theory is too idealized for direct application in such a design, it can be expected that the qualitative insights it provides will be useful.

Zimin and Lyakhov [1] were the first to discover that the non-magnetic laminar wall plume exhibits similarity within the context of the Boussinesq-boundary layer approximation. They compared a numerical solution for $Pr = 7$ with experimental data in water and found excellent agreement. Liburdy and Faeth [2] independently discovered a somewhat more general similarity transformation which allows for certain fluid property variations. They presented numerical solutions for $Pr = 0.01$, 0.1, 0.7, 1, 10, and 100. Jaluria and Gebhart [3], unaware of [2], treated the constant property wall plume using a more convenient set of boundary conditions and reported numerical solutions covering the same range of Prandtl number. These studies showed that compared to an unconfined line plume of twice the heat flux, the wall plume has a lower peak velocity, a higher maximum temperature, and a greater thickness. These deviations become more pronounced as Pr increases.

Grella and Faeth [4] reported an experimental investigation of a turbulent wall plume in air.

ANALYSIS

A horizontal line heat source is assumed to lie along the surface of a vertical adiabatic plate. The adjacent body of fluid is unstratified and is subject to a uniform gravitational force. A right-handed rectangular coordinate system is chosen so that the heat source lies along the z-axis, the positive x-axis points upward, and the positive y-axis extends into the fluid. The applied magnetic field is primarily in the y-direction and is a function only of x. There is no externally imposed electric field and the plate is electrically insulated from the fluid. The currents induced by the fluid motion close through a distant electrical circuit of perfect conductivity. Ohm's Law applies to the fluid and the magnetic Reynolds number is assumed small so that distortions of the applied magnetic field can be neglected. The variation of temperature is assumed to be small enough that density variations can be neglected, except in the buoyancy term of the vertical momentum equation, and all other fluid properties can be treated as constants. With viscous and Joulean dissipation neglected, the laminar boundary layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \alpha g\theta - \frac{\sigma}{\rho}B^2 u \tag{2}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \kappa\frac{\partial^2 \theta}{\partial y^2}. \tag{3}$$

Following Jaluria and Gebhart [3], the following boundary conditions are imposed:

$$u(x, y = 0) = 0 \tag{4}$$

$$v(x, y = 0) = 0 \tag{5}$$

$$u(x, y = \infty) = 0 \tag{6}$$

$$\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = 0 \tag{7}$$

$$\theta(x, y = 0) = \theta_w = Nx^n. \tag{8}$$

Equations (4), (5), and (7) reflect the non-slip, impermeable, adiabatic nature of the wall. Equation (6) says that the vertical velocity induced by the heat source vanishes far from the plate. The power-law variation of wall temperature specified by equation (8) yields consistent similarity solutions and allows the requirement of zero temperature excess far from the wall to be dropped as redundant, thus simplifying the numerical computations.

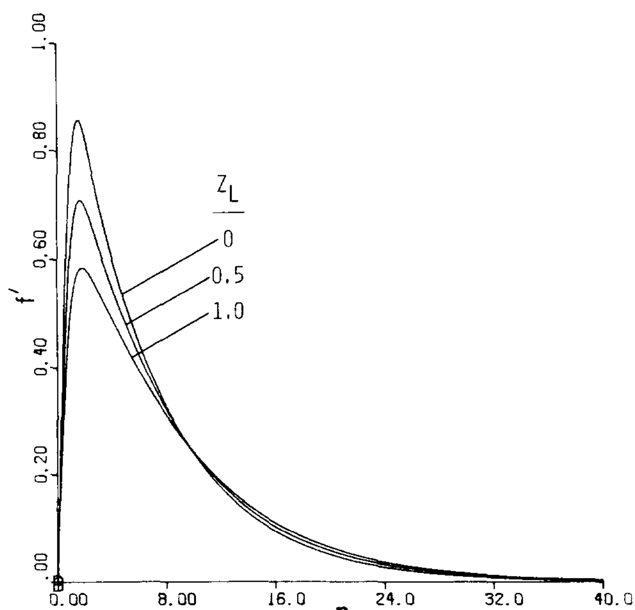

FIG. 1. Velocity similarity functions for $Pr = 0.01$, $Z_L = 0, 0.5, 1$.

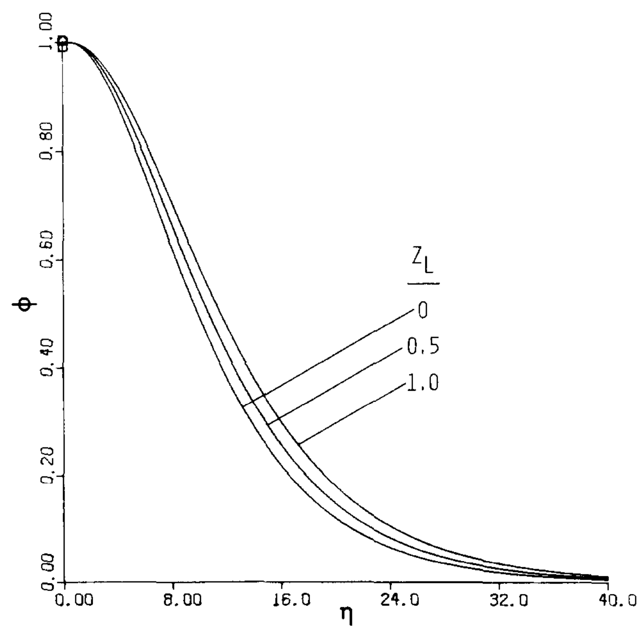

FIG. 2. Temperature similarity functions for $Pr = 0.01$, $Z_L = 0, 0.5, 1$.

In order to satisfy identically the continuity equation, a stream function is defined with the positive sign associated with u. The following transformations are then introduced:

$$\eta = \left[\frac{\alpha g N}{4\nu^2}\right]^{1/4} x^{-2/5} y \tag{9}$$

$$\psi = [64\alpha g \nu^2 N]^{1/4} x^{3/5} f(\eta) \tag{10}$$

$$\theta = N x^n \phi(\eta). \tag{11}$$

Because the wall is adiabatic and vertical conduction is assumed negligible, the power dissipated by the heat source must be convected across each overlying horizontal plane. This condition allows the exponent n to be determined.

$$Q = \rho C_p \int_0^\infty u\theta \,\mathrm{d}y = \rho C_p N (64\alpha g N \nu^2)^{1/4} x^{(5n+3)/4} \int_0^\infty f'\theta \,\mathrm{d}\eta.$$

Since Q is not a function of x,

$$n = -3/5. \tag{12}$$

Defining

$$I = \int_0^\infty f'\phi \,\mathrm{d}\eta, \tag{13}$$

implies that

$$N = \left[\frac{Q^4}{64 g \alpha \rho^4 C_p^4 \nu^2 I^4}\right]^{1/5}. \tag{14}$$

When equations (9)–(14) are introduced into the vertical momentum equation (2), it is seen that similarity is possible only if the magnetic field has the following variation:

$$B = B_0 x^{-2/5}. \tag{15}$$

In a strict sense such a field is inconsistent with Ampere's Law in the MHD approximation (as is the neglect of the induced field). Geometrically speaking, the field lines must exhibit some curvature. Nevertheless, the necessary curvature is small and the required field can be approximated

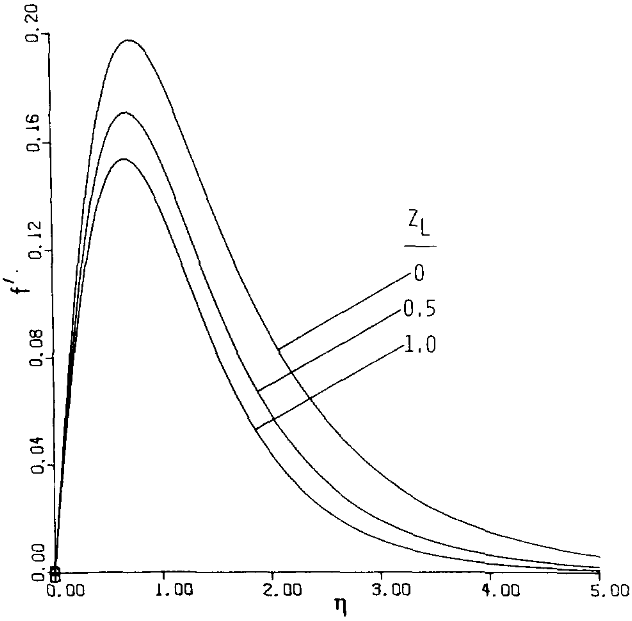

FIG. 3. Velocity similarity functions for $Pr = 10.0$, $Z_L = 0, 0.5, 1$.

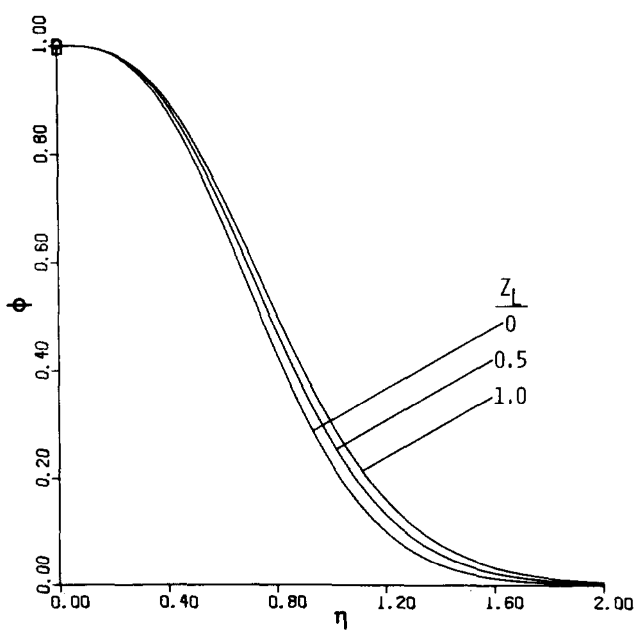

FIG. 4. Temperature Similarity Functions for $Pr = 10.0$, $Z_L = 0, 0.5, 1$.

very accurately in the region of non-negligible velocities. Equations (9)–(15) transform equations (2)–(8) into the following set of coupled nonlinear ordinary differential equations:

$$f''' - \tfrac{4}{5}f'f' + \tfrac{12}{5}ff'' + \phi - Z_L f' = 0 \quad (16)$$

$$\phi'' + \tfrac{12}{5}Pr(f\phi)' = 0 \quad (17)$$

$$f(0) = 0 \quad (18)$$

$$f'(0) = 0 \quad (19)$$

$$f'(\infty) = 0 \quad (20)$$

$$\phi'(0) = 0 \quad (21)$$

$$\phi(0) = 1, \quad (22)$$

where $Z_L = 2(\sigma/\rho)B_0^2(\alpha g N)^{-1/2}$ is the Lykoudis number, a measure of the ratio of the ponderomotive force to the square root of the buoyancy force times the inertial force, and Pr is the Prandtl number.

The solution of equations (16)–(22) was obtained numerically using a fourth-order Runge–Kutta program. Satisfaction of boundary condition (20) was approached asymptotically using the iterative algorithm of Nachtsheim and Swigert [5]. Solutions were calculated for Lykoudis numbers of 0, 0.5, and 1, as the Prandtl number ranged from 0.01 to 100. Representative results are presented in Table 1 and Figs. 1–4.

The physical velocities are related to the similarity variables by:

$$u = \left(\frac{4}{\nu}\right)^{1/5}\left(\frac{\alpha g Q}{\rho C_p I}\right)^{2/5} x^{1/5} f' \quad (23)$$

$$v = \left[\frac{64\alpha g \nu^2 Q}{\rho C_p I}\right]^{1/5} x^{-2/5}\left[\tfrac{2}{5}\eta f' - \tfrac{3}{5}f\right]. \quad (24)$$

Table 1. Wall plume parameters

Pr	Z_L	η_{max}	$f''(0)$	$f(\infty)$	I
100	0	11.0	0.37672	0.20636	0.021001
	0.5	9.0	0.35254	0.11137	0.019808
	1.0	11.0	0.33789	0.084434	0.019076
10	0	11.0	0.61059	0.38320	0.10675
	0.5	10.0	0.55871	0.28680	0.098935
	1.0	8.0	0.52173	0.23742	0.093212
1	0	11.0	0.88725	0.77581	0.44712
	0.5	11.0	0.79080	0.70000	0.41325
	1.0	11.0	0.71436	0.63751	0.38400
0.1	0	24.0	1.1380	2.1298	1.4893
	0.5	21.0	0.97417	2.0016	1.3903
	1.0	27.0	0.84633	1.8862	1.2972
0.01	0	102.0	1.3033	6.62	4.72
	0.5	56.0	1.0733	6.27	4.42
	1.0	49.0	0.90312	5.85	4.13

The wall shear stress is:

$$\tau = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \left(\frac{\rho\mu}{4}\right)^{1/5}\left(\frac{\alpha g Q}{C_p I}\right)^{3/5} x^{-1/5} f''(0), \qquad (25)$$

and the vertical mass flow rate is:

$$\dot{m} = \rho \int_0^\infty u\,dy = \left[\frac{64\alpha g \rho^2 \mu^2 Q}{C_p I}\right]^{1/5} f(\infty) x^{3/5}. \qquad (26)$$

As a consequence of the boundary layer approximation, the solution is not valid too near the heat source or for negative x. At some distance above the heat source the plume may become turbulent, but it is expected that stabilizing effects of the wall and the magnetic field should combine to produce an extended laminar regime.

RESULTS AND DISCUSSION

The effect of the magnetic field enters the expressions for velocity and temperature in three ways. As the figures illustrate, the profiles of f' and ϕ are altered as Z_L increases. This leads to a change in the value of I which in turn changes the coefficients in equations (11) and (23)–(26). The change in I also alters the η value of a given physical location as shown by equations (9) and (14).

Figure 1 shows that for $Pr = 0.01$ (typical of liquid metals), the peak value of f' decreases as Z_L increases. Figure 2 indicates that the corresponding temperature similarity function ϕ becomes wider as Z_L increases. As expected, the velocity and temperature fields are of comparable width for low Pr natural convection.

In Figs. 3 and 4 profiles are presented for $Pr = 10.0$, a value representative of molten salts [6]. Figure 3 shows that the peak value of f' is decreased by the magnetic field, and Fig. 4 indicates that ϕ becomes wider. In this case it is seen that the velocity profile is much wider than the temperature profile.

Solutions were obtained for the values of Pr and Z_L listed in Table 1, and in all cases an increase in Z_L causes a reduction in the peak value of f' and a broadening of ϕ. The percentage reduction of the peak value of f' caused by an increase in Z_L from 0 to 1 rises smoothly from 20% at $Pr = 100$ to 32% at $Pr = 0.01$. Table 1 shows that the values of $f''(0)$, $f(\infty)$, and I all decrease with increasing Pr and Z_L. Similar trends occur in the case of the unconfined magneto fluid mechanic line plume [7]. Although it is not immediately apparent, further consideration of equations (23), (25), and (26) shows that for any given fluid, the peak vertical velocity, wall shear stress, and vertical mass flow rate are all reduced by increasing Z_L. Wall temperature, on the other hand, increases with Z_L. The percentage reduction of the peak vertical velocity due to a change in Z_L from 0 to 1 increases monotonically from 17% at $Pr = 100$ to 28% at $Pr = 0.01$. The corresponding reduction in wall shear stress increases steadily from 5% at $Pr = 100$ to 25% at $Pr = 0.01$.

As in the case of the unconfined magneto fluid mechanic line plume, laboratory experiments using liquid metals and molten salts are feasible for the wall plume [7]. The solution should be valid from a level somewhat above the heat source to the level at which transition to turbulence begins.

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